Consider the following two sequences: **(i)** $2, 4, 6, 8, ...$ **(ii)** $2, 4, 8, 16, ...$

Explain how to calculate t_5 and t_6 for each sequence, and calculate the values for each term.

1 Geometric Sequences

A sequence in which each term is calculated by **multiplying** the previous term by a constant is called a **geometric** sequence.

For example, $5, 10, 20, 40, ...$ is a geometric sequence.

The value of the constant can be found by dividing any term of the sequence by the previous term:

$\frac{t_2}{t_1}$, or $\frac{t_5}{t_4}$, etc. In the example above, the constant is ____________.

We call the constant in a geometric sequence the **common ratio**:

$$\boxed{\textbf{common ratio} = \mathbf{r} = \frac{t_n}{t_{n-1}}, n \geq 2, n \in \mathbb{N}}$$

1.1 Example

Find the common ratio for each of the following sequences:

(a) $6, 12, 24, 48, ...$ (b) $-1, 5, -25, 125, ...$ (c) $-10, -5, -\frac{5}{2}, -\frac{5}{4}, ...$

1.2 Example

For the sequences below,

- State whether the sequence is arithmetic or geometric
- Find the common difference or common ratio
- Calculate the 5^{th} and 6^{th} terms for each sequence

(a) $8, 24, 72, 216, ...$ (b) $108, 72, 36, 0, ...$ (c) $-3, 2, -\frac{4}{3}, \frac{8}{9}, ...$

2 A Formula for the General Term of a Geometric Sequence

Consider the following sequence:

$8, 12, 18, 27, ...$

(a) State the common ratio

(b) Let $a =$ the first term of the sequence, and $r =$ the common ratio. Fill in the table:

t_1	First term = 8	$t_1 = a$
t_2	$8 * 1.5 = 12$	$t_2 = ar$
t_3	$8 * 1.5 * 1.5 = 18$	$t_2 = ar^2$
t_4	$8 * 1.5 * 1.5 *$ _____ $=$	$t_4 =$
t_5		$t_5 =$
t_n		$t_n =$

(c) The formula for the general term of a geometric sequence is $t_n =$

The Formula for the General Term of a Geometric Sequence

The formula for the general term of a geometric sequence is:

$$\boxed{t_n = ar^{n-1}}$$

or

$$\boxed{t_n = t_1 r^{n-1}}$$

where

t_n is the general term of the geometric sequence,

$a = t_1$ is the first term,

r is the common ratio, and

n is the position of the term being considered.

Geometric sequences follow this pattern:

$t_1, \quad t_2, \quad t_3, \quad t_4, \quad ..., \quad t_n$

$a, \quad ar, \quad ar^2, \quad ar^3, \quad ..., \quad ar^{n-1}$

2.1 Example

Determine the formula for the general term (and calculate the indicated term) for each sequence:

(a) $5, 15, 45, ...$ Calculate t_9.

(b) $\frac{1}{3}, -\frac{1}{6}, \frac{1}{12}, ...$ Calculate t_7.

3 Determining the Number of Terms in a Geometric Sequence

Russell and Brittany were trying to determine the number of terms in the sequence $32, 64, 128, ..., 16384$.

(a) Use the general term formula to write (and simplify!) an equation that could be used to determine the number of terms.

(b) Russell used the "guess and check" method to write each side of the equation from part a) with a common base. Use this method to determine a solution.

(c) Brittany found the intersection of two graphs to solve the equation. Use this method to determine a solution.

4 Geometric Means

The terms between two non-consecutive terms in a geometric sequence are called **geometric means**. As with finding arithmetic means, it is helpful to think of the two terms as the first and last terms of the sequence.

4.1 Example

Insert four geometric means between 81 and $\frac{1}{729}$.

5 Solving Problems When Both "a" and "r" Are Unknown

5.1 Example

Three consecutive terms of a geometric sequence are $x+3$, x, and $x-5$. Use the concept of a common ratio to determine the value of x and the three terms.

5.2 Example

The 4^{th} and 7^{th} terms of a geometric sequence are -54 and 1458, respectively. Use two equations to determine the value of the first term, the value of the common ratio, and to find the formula for the general term of the sequence.